7-6 Proportional Lengths

If two segments are **divided proportionally**, what do you think is true about each segment's component pieces?

They are in <u>proportion</u>!

If $\overline{AC}$ and $\overline{JL}$ are <u>divided proportionally</u>, then $\frac{AB}{BC} = \frac{JK}{KL}$.

Example 1 – Segments $\overline{AC}$ and $\overline{GH}$ are divided proportionally. Find AB.

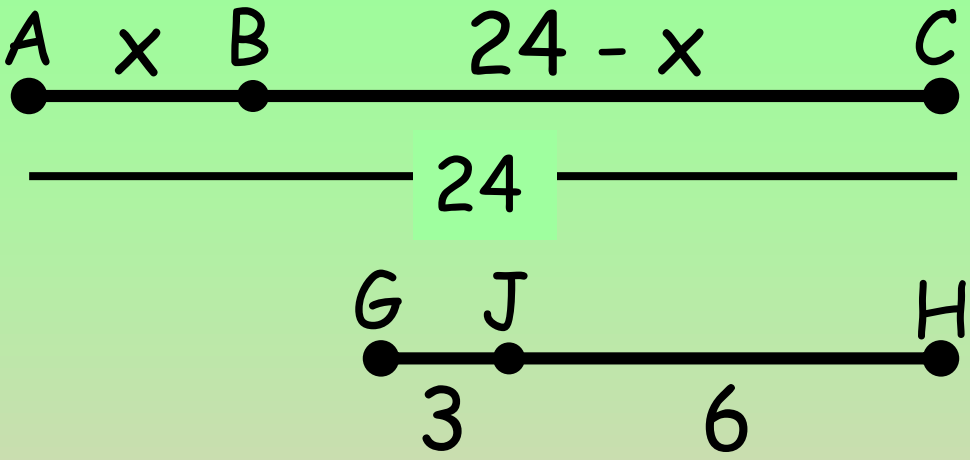
$$\frac{AB}{BC} = \frac{GJ}{JH}$$

$$\frac{x}{24-x} = \frac{3}{6}$$

$$6x = 72 - 3x$$

$$9x = 72$$

$$x = 8$$

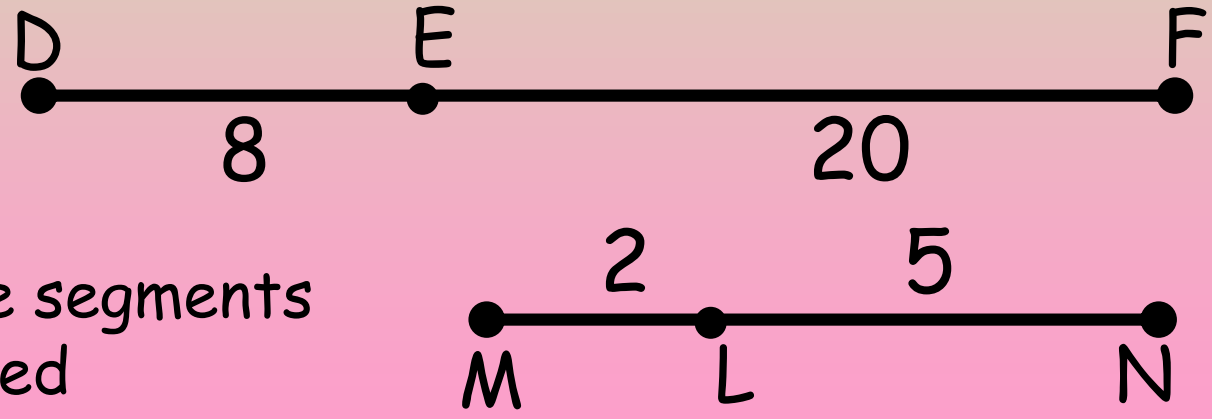


Example 2 – Are segments $\overline{DF}$ and $\overline{MN}$ divided proportionally?

$$\frac{DE}{EF} = \frac{ML}{LN}?$$

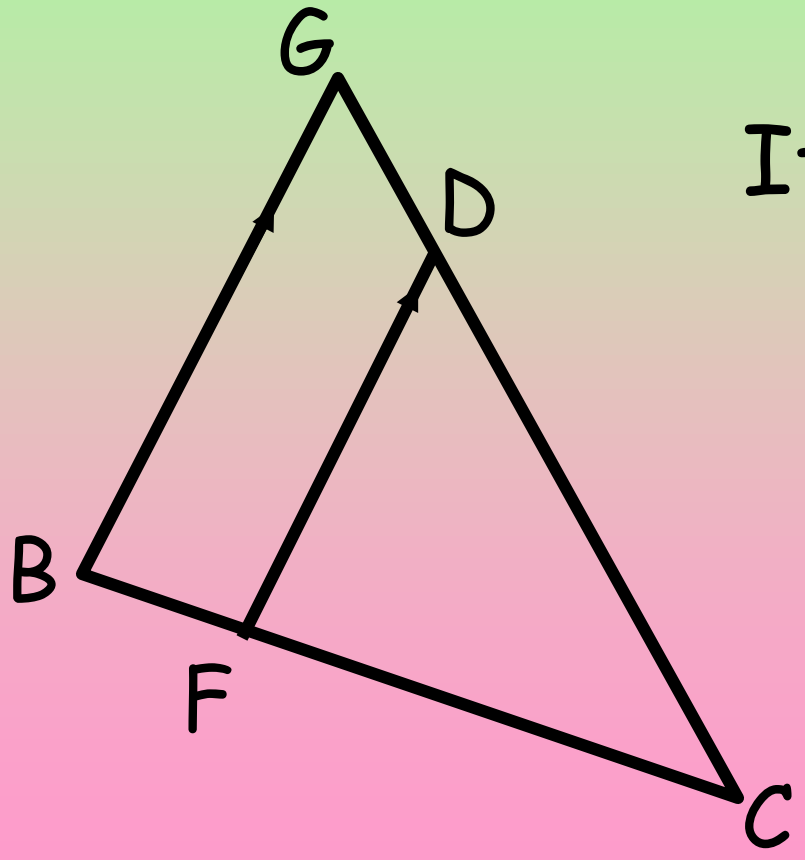
$$\frac{8}{20} = \frac{2}{5}?$$

Yes. The segments are divided proportionally.



Triangle Proportionality Theorem

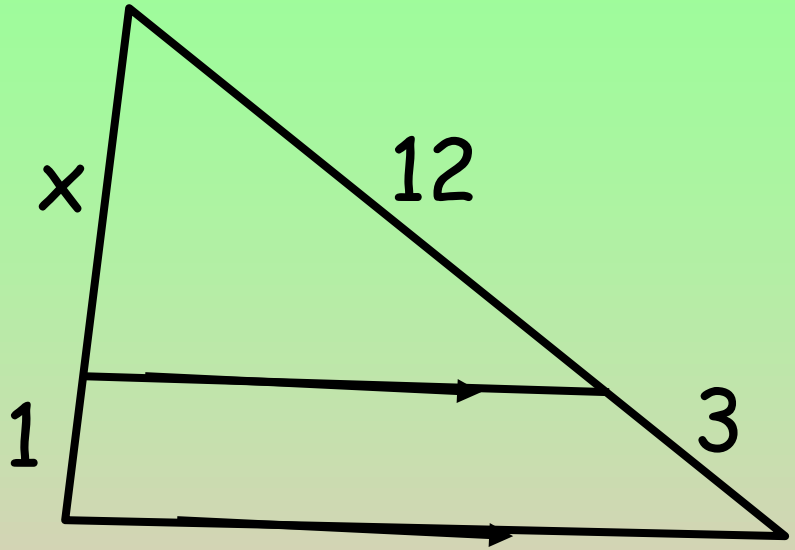
If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.



If $\overline{GB} \parallel \overline{DF}$,

then $\frac{GD}{DC} = \frac{BF}{FC}$

Example 3 -

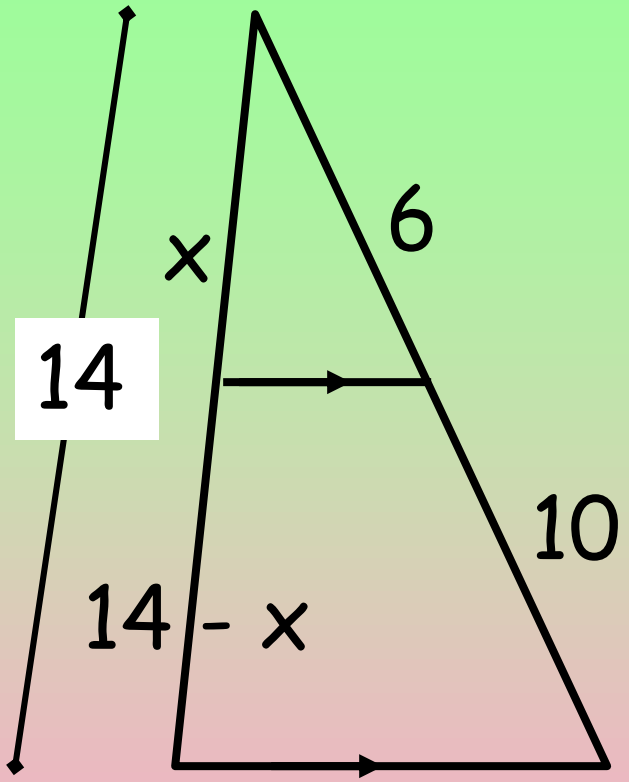


$$\frac{x}{1} = \frac{12}{3}$$

$$3x = 12$$

$$x = 4$$

Example 4 -



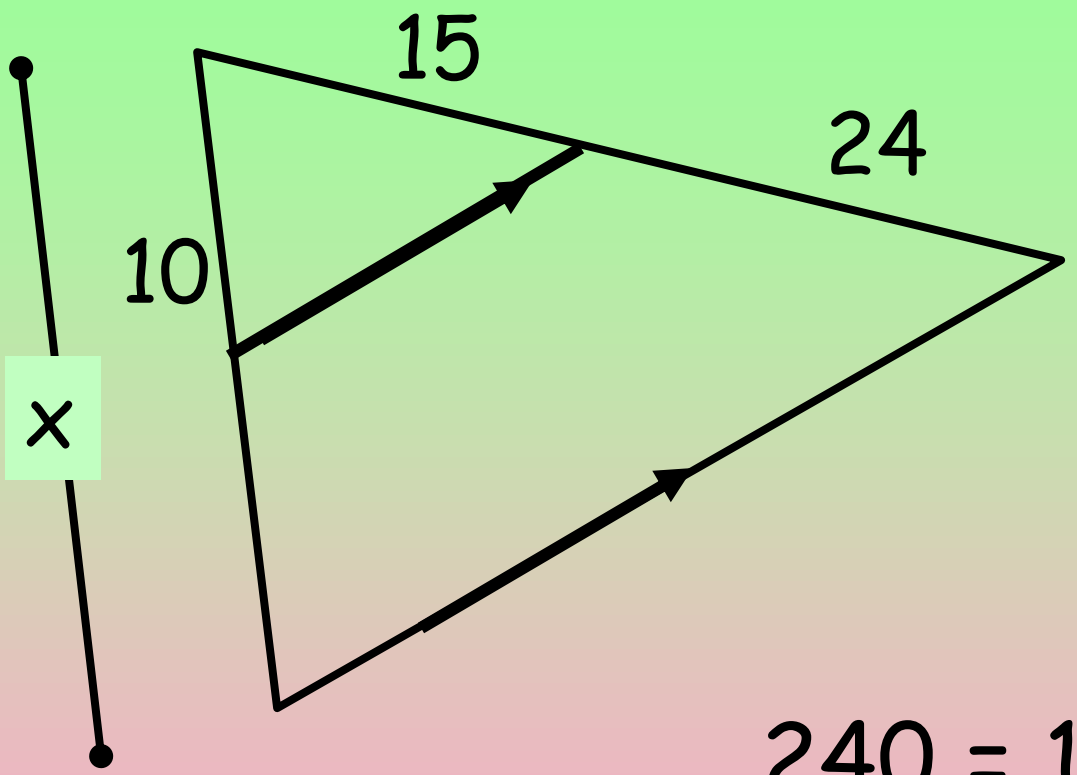
$$\frac{x}{14-x} = \frac{6}{10}$$

$$10x = 84 - 6x$$

$$16x = 84$$

$$x = 5.25$$

Example 5 -



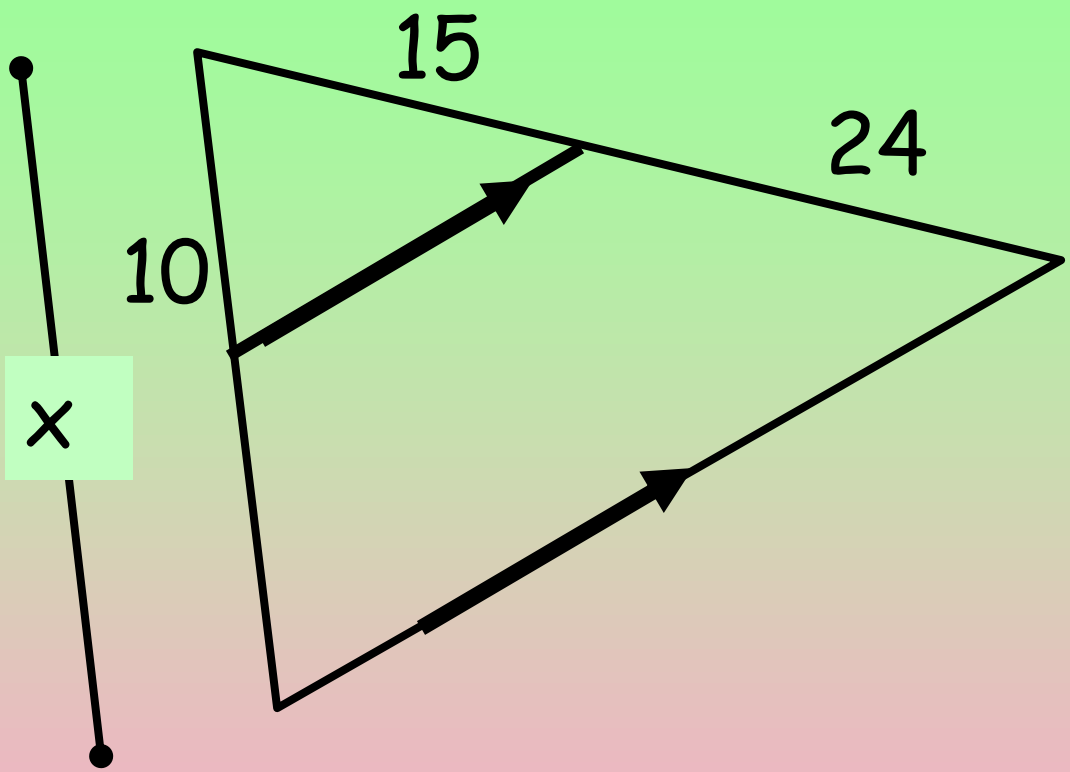
$$\frac{10}{x-10} = \frac{15}{24}$$

$240 = 15x - 150$

$390 = 15x$

$x = 26$

Example 5 – revisited.



$$\frac{10}{x} = \frac{15}{39}$$

$$390 = 15x$$

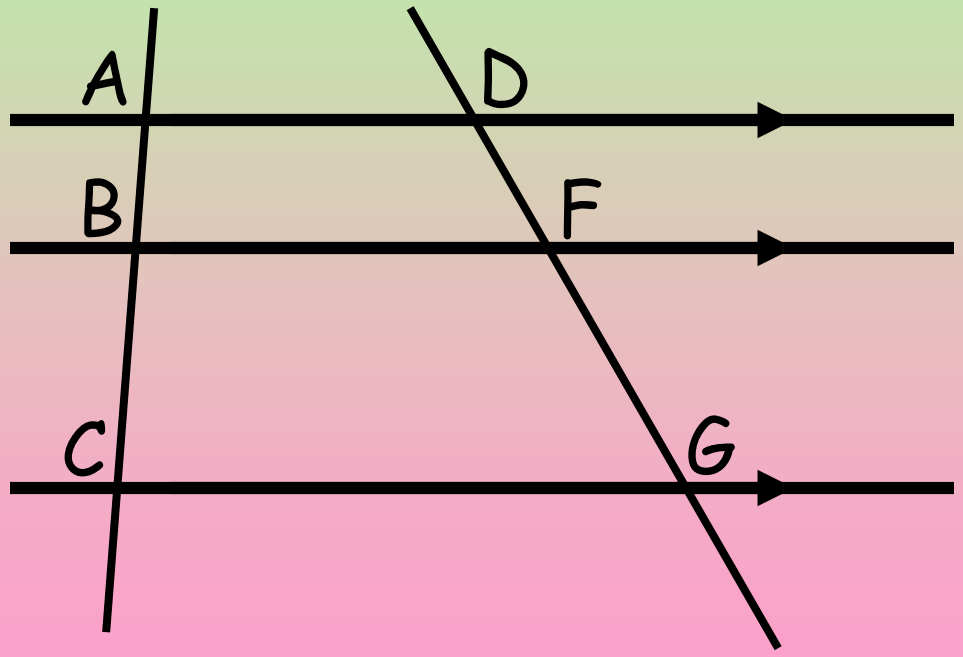
$$x = 26$$

Corollary:

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

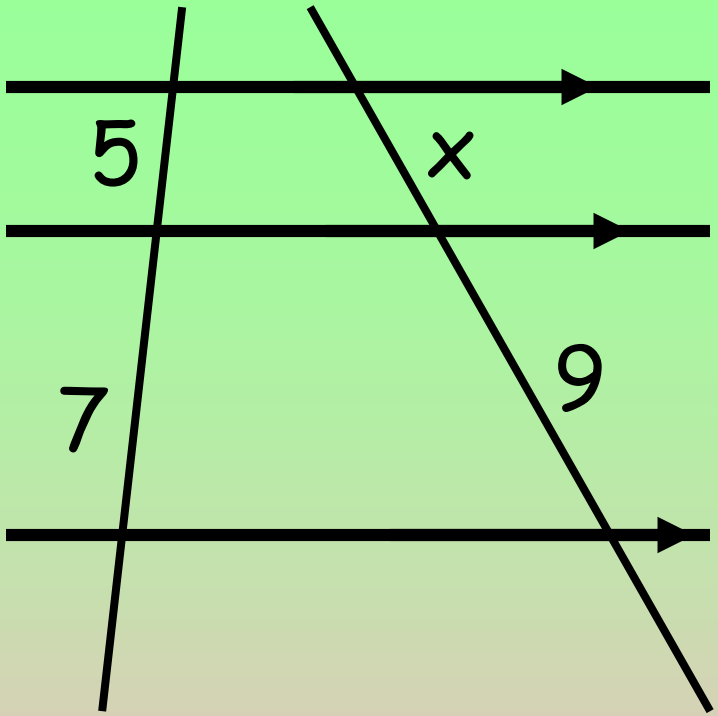
If $\overline{AD} \parallel \overline{BF} \parallel \overline{CG}$,

then $\frac{AB}{BC} = \frac{DF}{FG}$



Example 6 -

$$\frac{5}{7} = \frac{x}{9}$$

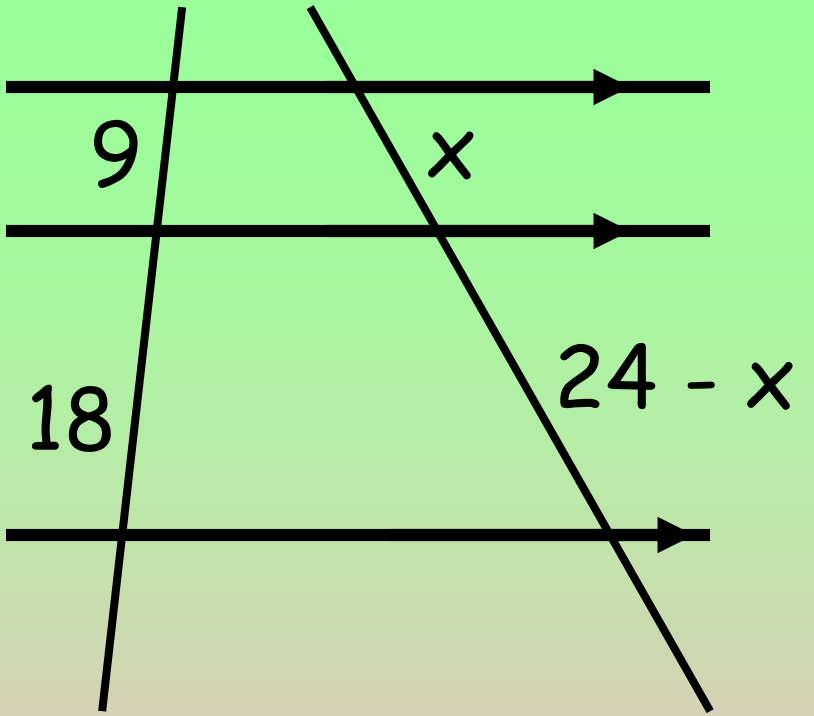


$$45 = 7x$$

$$x = \frac{45}{7}$$

Example 7 -

$$\frac{9}{18} = \frac{x}{24 - x}$$



$$216 - 9x = 18x$$

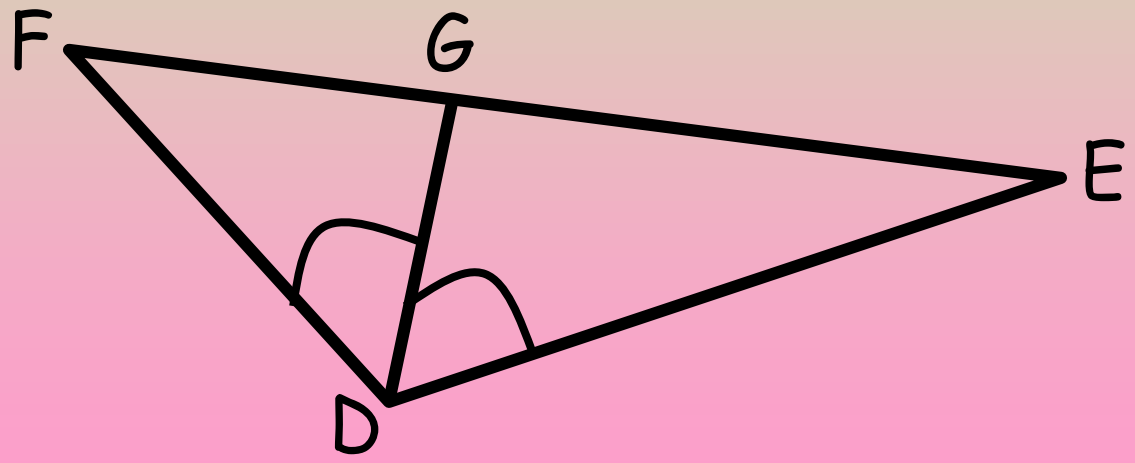
$$216 = 27x$$

$$x = 8$$

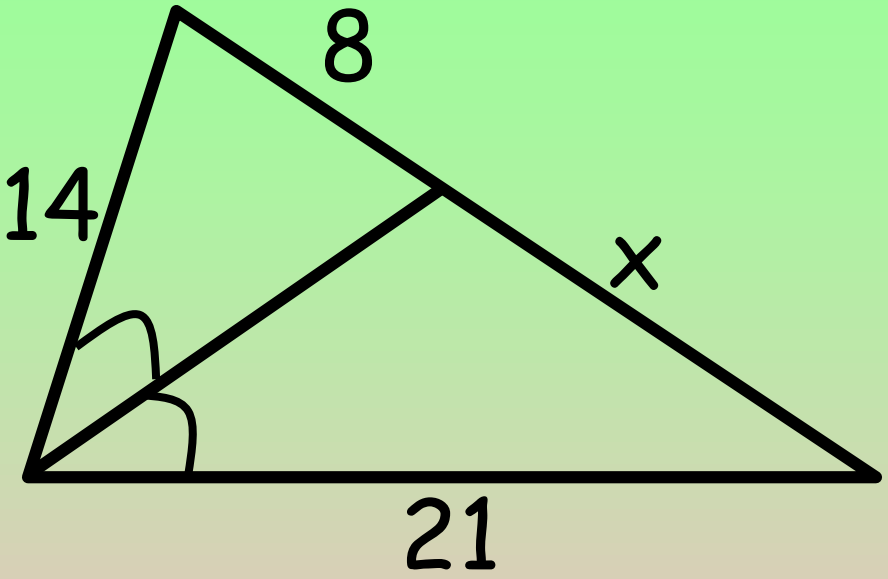
Triangle Angle-Bisector Theorem:

If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

If $\overrightarrow{DG}$ bisects $\angle FDE$ then $\frac{GF}{GE} = \frac{DF}{DE}$



Example 8 -

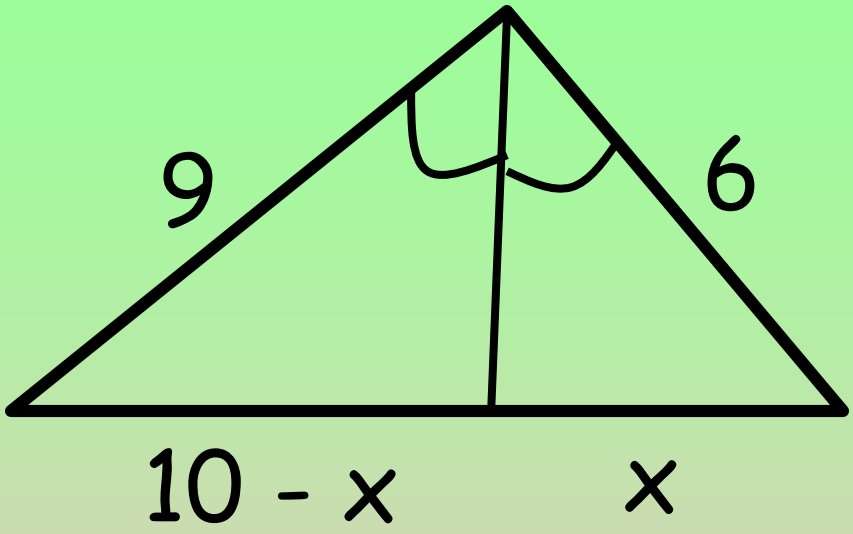


$$\frac{8}{x} = \frac{14}{21}$$

$$168 = 14x$$

$$x = 12$$

Example 9 -



$$\frac{10-x}{x} = \frac{9}{6}$$

$9x = 60 - 6x$

$15x = 60$

$x = 4$